RI Data Processing System for Nuclear Medicine (Report 15) Canonical Model and It's Transform in Compartmental Analysis

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Compartmental systems are mathematical models used to describe measurements made with radioactive isotopes (RI) of physiological substance. In the Nuclear Medicine, compartmental analysis is one of important methods to analyze RI tracer kinetics. Because useful parameter for the aid of RI diagnosis can be extracted from this analysis.

The present study was attempted to determine the canonical form in 3 compartmental system and to transform to mammillary and catenary models. The Laplace transform of the measured compartment Q can be written as

$$1/Q=s+a+bs/(s+\sigma_1)+bs(\sigma_1-k\sqrt{\sigma_1\sigma_2})/(s+\sigma_1)(s+\sigma_2)$$

From this equation, the canonical form of 3 compartmental system was obtained and it's rate constants matrix was given as,

$$\lambda=\begin{pmatrix} s+a+b & -k\sqrt{\sigma_1\sigma_2} & -\sigma_2 \\ -b & s+\sigma_1 & 0 \\ 0 & -(\sigma_1-k\sqrt{\sigma_1\sigma_2}) & s+\sigma_2 \end{pmatrix}$$

The transform to catenary or mammillary models can be obtained by

$\lambda'=P\lambda P^{-1}$ and

in mammillary model,

$$\lambda'=\begin{pmatrix} s+a+b & -\sigma_1 & -\sigma_2 \\ -b\dfrac{k\sqrt{\sigma_1\sigma_2}-\sigma_2}{\sigma_1-\sigma_2} & s+\sigma_1 & 0 \\ -b\dfrac{\sigma_1-k\sqrt{\sigma_1\sigma_2}}{c_1-\sigma_2} & 0 & s+\sigma_2 \end{pmatrix}$$

in catenary model,

$$\lambda'=\begin{pmatrix} s+a+b & -k\sqrt{\sigma_1\sigma_2} & 0 \\ -b & s+\dfrac{(\sigma_1-k\sqrt{\sigma_1\sigma_2})(k\sqrt{\sigma_1\sigma_2}-\sigma_2)}{k\sqrt{\sigma_1\sigma_2}} & -\dfrac{\sigma_1\sigma_2}{k\sqrt{\sigma_1\sigma_2}} \\ 0 & -\dfrac{\sigma_1\sigma_2+k\sqrt{\sigma_1\sigma_2}(\sigma_1+\sigma_2)}{k\sqrt{\sigma_1\sigma_2}}s & +\dfrac{\sigma_1\sigma_2}{k\sqrt{\sigma_1\sigma_2}} \end{pmatrix}$$