A Mathematical Model of Radiocardiogram and its Analog Simulation Circuit

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Radiocardiogram is considered as a representation of hemodynamics of inflow into and outflow from right and left hearts of injected isotope. In this paper, it is shown that an analog computer analysis of the hemodynamics simulated by an appropriate mathematical model makes to be able to quantify cardiac output and equivalent volumes of right and left hearts, pulmonary and body blood vessels.

Transportation processes of injected isotope I (μCi) neglecting heart-beat can be approximately represented by following fundamental equations:

$$V_r c_r(t) = F_i \int_0^t c_i(t)dt + F\int_0^t c_b(t-\tau_b)dt - F\int_0^t c_r(t)dt$$

$$V_p c_p(t) = F\int_0^t c_r(t)dt - F\int_0^t c_p(t)dt$$

$$V_l c_l(t) = F\int_0^t c_p(t-\tau_p)dt - F\int_0^t c_l(t)dt$$

$$V_b c_b(t) = F\int_0^t c_l(t)dt - F\int_0^t c_b(t)dt$$

where F (ml/sec) is a mean blood flow rate, V's (ml) and c(t)'s (μCi/ml) represent equivalent volumes and isotope concentration of right and left hearts, pulmonary and body blood vessels, and τ's (sec) represent transportation lags in pulmonary and body systems.

Transportation process at the injected part

$$V_i C_i(t) = \int_0^t \frac{I}{\tau}dt - F_i\int_0^t ci(t)dt$$

These equations construct a mathematical model of hemodynamics transporting injected isotope.

Cardiac output can be calculated from following equations by using parameter values T's and τ's and mean concentration C(∞) in steady state.

$$V = V_r + V_p + F\tau_p + V_c + V_b + F\tau_b = I/c(\infty)$$

$$Fc(\infty) = \frac{IF}{V_r + V_p + F\tau_p + V_c + V_b + F\tau_b} = \frac{I}{T_r + T_p + \tau_p + T_l + T_b + \tau_b}$$

where $T_r = V_r/F$, $T_p = V_p/F$, $T_l = V_l/F$ and $T_b = V_b/F$ are time constants of four blood vessels of right heart, pulmonary, left heart and body systems, and $(V_p + F\tau_p)$ and $(V_b + F\tau_b)$ represent pulmonary and body blood volumes.

A generalized mathematical model of cases where are a shunt of left heart to right heart and in inverse shunt of right heart to left heart was developed and a generalized hemodynamics simulation circuit was constructed by using an analog computer on the basis of the above model.

Simulated radiocardiograms obtained from the simulation circuit showed good coincidence with measured ones and calculated cardiac outputs for 20 patients were almost equal to measured cardiac outputs by Fick method.

Heart Pool Scintigram

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In 1958, Rejali et al introduced the radioisotope heart pool scan utilizing ^{131}I labeled human serum albmin. It has been helpful in the differential diagnosis of pericardiol effusion, vascular aneurysms and mediastinal masses.