Interative Correction for a Finite Resolving Power of the Collimator in Scintiscanning

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The most inherent problem in scintiscanning is the contradiction between resolving power of the collimator and detection efficiency. Indeed, the best spatial resolution would be obtained with the detector of zero sensitivity (=infinite collimation). If we can correct the distortion in a scingram due to a finite resolution of the collimator, we can seek a better compromise between the two opposing factors.

A correction method described here is based upon the iterative approximation that has been used in correcting a distortion in β- or γ-ray spectra due to a finite energy resolution of the detectors.(1)

For simplicity, we consider firstly a case of linear (profile) scanning. In this case the distribution of a radionuclide in vivo is assumed to be a line source which may be divided into n sections of an unit length of different activities. The activity of the ith section is noted as ΔIi. The length of a section needs to be smaller than a field of vision of the collimator. The resolving power of the collimator is experimentally obtained by moving ΔIi along a long axis of the body and would give a symmetric distribution having a peak where ΔIi is placed coaxially with the crystal axis. Thus, the resolving power is written in a matrix form, $|A|$, in which an element A_0 denotes a fractional contribution from ΔIi placed on the crystal axis, and the elements A_1 and A_1' are those from the ΔIi displaced from the axis by an unit length in an either side, and so on. For the following calculation, the $|A|$ should be normalized to an unit activity, that is $\Sigma A_K=1$.

If we represent an observed profile scintigram by $|X|$ in which an element, Xi, corresponds to the counting-rate when the collimated crystal is placed coaxially with the ith section, the Xi is expressed in terms of the $|A|$ and $|\Delta I|$ as follows:

$$X_i=A_{k'}\Delta I_{i-k}+\cdots+A_{I'}\Delta I_{i-1}+A_0\Delta I_i+A_1\Delta I_{i+1}+\cdots+A_k\Delta I_{i+k} \quad (1)$$

where $A_0\Delta I_i$ is a contribution of ΔI_i on the crystal axis, and $A_1\Delta I_{i+1}$ is that of ΔI_{i+1} positioning at the next section to the ΔI_i, and so on.

If we use a matrix form for all Xi, the relationship of eq.(1) can be rewritten:

$$\begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{vmatrix} = \underbrace{\begin{vmatrix} A_{k'}\cdots\cdots A_{1'}\ A_0\ A_1\cdots\cdots A_k\ 0\ 0\cdots 0 \\ 0\ A_{k'}\cdots\cdots A_{1'}\ A_0\ A_1\cdots\cdots A_k\ 0\cdots 0 \\ 0\ 0\ A_{k'}\cdots A_{1'}\ A_0\ A_1\cdots A_k\ 0\cdots 0 \\ \cdots\cdots \\ \cdots\cdots \end{vmatrix}}_{n} \begin{vmatrix} \Delta I_1 \\ \Delta I_2 \\ \vdots \\ \Delta I_n \end{vmatrix} \quad \cdots\cdots (2)$$

Here, the iterative approximation allows to approach the true distribution, the $|\Delta I|$, by knowing the $|X|$ and the $|A|$ without knowledge of $|A|^{-1}$, the inverse matrix of $|A|$.

$$|X^1|=|X|+(|X|-|A||X|) \quad \cdots\cdots(3)$$

The second one,

$$|X^2|=|X^1|+(|X|-|A||X^1|) \quad \cdots\cdots(3)'$$

The ith one,

$$|X^i|=|X^{i-1}|+(|X|-|A||X^{i-1}|) \quad \cdots(3)''$$

When $|A||X^i|$ is equal to $|X|$, $|X^i|$ should be equal to $|\Delta I|$.

Since the above equations are written in the matrix form, the calculation of an element X (the first approximation) is explained as an example:

$$X_i^1=X_i+\{X_i-(A_{k'}X_{i-k}+\cdots+A_{1'}X_{i-k}+A_0X_i+A_1X_{i+1}+\cdots A_kX_{i+k})\}$$
$$=X_i+\{X_i-\sum_k A_kX_{i+k}\} \quad \cdots\cdots(4)$$

The calculation of eq.(4) must be made for all X. Usually the number of elements in |A|, that is 2K+1, is much smaller than n.

The approximation should be stopped when $\sum AX_i$ and X_i agree to within the statistical standard deviation $\sqrt{X_i}$. For all n sections, the limit of the approximation is derived:

$$\sum_{i=1}^{n} \frac{\{X_i - \sum A_k X^i_{i+k}\}_2}{X_i} < n \quad \cdots\cdots(5)$$

The above method can be extended to the correction for the distortion in a two-dimentional scanning (area-scanning). In this case, ΔI stands for the activity of an unit area,

* For detailed mathematical discussions for the condition of convergence of eq.(3″), the reference (1) is referred.

S, in an area source, and $||\Delta I||$ for the true distribution to be determined which consists of n×m unit areas. Resolving power of a collimator should be expressed as a two-dimentional broadening, ||A||, by which the $||\Delta I||$ is distorted. Elements Akl of the ||A|| have such characteristics that A_{00} denotes a fractional contribution of ΔI that is positioned on the crystal axis, and that Akl are those of ΔI at various positions from the axis in the area source. ΣAkl should be normalized to unity like in the case of profile scanning.

The observed scintigram ||X|| is also written in a matrix form. An element X of ||X|| corresponds to a counting-rate when the crystal is coaxially positioned with the unit area of ΔS.

The iterative calculation is proceeded as follows: for simplicity, calculation of an element X is shown;

The first approximation,

$$X^1_{ij} = X_{ij} + \{X_{ij} - \sum_{kl} A_{kl} \cdot X_{i+k,\, j+l}\} \quad \cdots\cdots(6)$$

Where $\sum A_{kl} \cdot X_{i+k\ j+l}$ means that the multiplication starts from $A_{00} \cdot X_{ij}$ and then A_{kl} that corresponds to a certain distance from A_{00} should be multiplied by $X_{i+k\ j+l}$ that corresponds to an identical distance from X_{ij}, and the same process is repeated for all A_{kl}. The calculation of eq. (6) must be performed for all X_{ij}.

Similarly the ith approximation,

$$X^i_{ij} = X^{i-1}_{ij} + \{X_{ij} - \sum A_{kl} \cdot X^{i-1}_{i+k\ j+l}\} \quad \cdots(6)'$$

The iteration should be stopped when the following condition is satisfied:

$$\sum \frac{\{X_{ij} - \sum A_{kl} \cdot X^i_{i+k\ j+l}\}^2}{X_{ij}} \leq n \times m$$

High-resolution scintigram with high-energy γ-emitters (^{59}Fe, ^{60}Co, and ^{47}Ca etc.) that has been difficult to obtain by any conventional methods might be obtainable with this correction. Moreover, this method could be used in analyzing scintigrams with smaller amount of tracers because the data are available in digital form.

Strictly speaking, the method is valid only under the assumption that the |A| does not change much with depth of the source in tissue. Consequently, it requires that efficiency of a detector used should have little dependency on the source depth. However, this condition can often be satisfied with the organ scanning. Calculation for the correction on an area scanning is so vast that use of a digital computer is unavoidable.

An application of this method to the actual area scanning is now in progress and the results will be reported later.

REFERENCE

(1) L.D. Skarsgard, H.E. Johns and L.E.S. Green, Iterative Response Correction for a Scintillation Spectrometer, Radiation Research 14, 126, (1961)

(2) T.A. Iinuma et. al. to be published.